## A Novel Unified Analytical Model for Broadcast Protocols in Multi-Hop Cognitive Radio Ad Hoc Networks

#### Yi Song, Jiang (Linda) Xie, and Xudong Wang

Abstract—Broadcast is an important operation in wireless ad hoc networks where control information is usually propagated as broadcasts for the realization of most networking protocols. In traditional ad hoc networks, since the spectrum availability is uniform, broadcasts are delivered via a common channel which can be heard by all users in a network. However, in cognitive radio (CR) ad hoc networks, different unlicensed users may acquire different available channels depending on the locations and traffic of licensed users. This non-uniform channel availability leads to several significant differences and causes unique challenges when analyzing the performance of broadcast protocols in CR ad hoc networks. In this paper, a novel unified analytical model is proposed to address these challenges. Our proposed analytical model can be applied to any broadcast protocol with any CR network topology. We propose to decompose an intricate network into several simple networks which are tractable for analysis. We also propose systematic methodologies for such decomposition. Results from both the hardware implementation and software simulation validate the analysis well. To the best of our knowledge, this is the first analytical work on the performance analysis of broadcast protocols for multi-hop CR ad hoc networks.

<sup>12</sup> Index Terms—Cognitive radio ad hoc networks, unified analytical model, network-wide broadcast, channel hopping, non-uniform <sup>13</sup> channel availability

#### 14 **1** INTRODUCTION

THE rapid growth of wireless devices has led to a dra-15 **F** matic increase in the demand of the radio spectrum. 16 17 However, according to the Federal Communications Commission (FCC), almost all the radio spectrum for wire-18 19 less communications has already been allocated. To alle-20 viate the spectrum scarcity problem, FCC has suggested a <sup>21</sup> new paradigm for dynamically accessing the allocated spec-22 trum [1]. Cognitive radio (CR) technology has emerged as 23 a promising solution to realize dynamic spectrum access 24 (DSA) [2]. Unlicensed users (or, secondary users) equipped 25 with the CR technology can form a CR infrastructure-<sup>26</sup> based network or a CR ad hoc network to opportunistically 27 exploit the licensed channels which are not used by licensed 28 users (or, primary users) [3].

In CR ad hoc networks, control information exchange
among nodes, such as channel availability and routing
information, is often sent out as network-wide broadcasts
(i.e., messages that are sent to all other nodes in a network) [4]. Such control information exchange is crucial for the realization of most networking protocols. In addition,

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some exigent data packets such as emergency messages and alarm signals are also delivered as network-wide broadcasts [5]. Therefore, broadcast is an essential operation in CR ad hoc networks.

Even though the broadcasting issue has been stud- 39 ied extensively in traditional mobile ad hoc networks 40 (MANETs) [6]–[10], research on broadcasting in multi-hop 41 CR ad hoc networks is still in its infant stage. There 42 are a few papers addressing the broadcasting issue in 43 multi-hop CR ad hoc networks [11]–[14]. However, these proposals mainly focus on broadcast protocol designs. 45 The performance analysis of these proposed protocols is 46 simulation-based. Thus, the analytical relationship between 47 these proposals and their performance is not known. More importantly, without analytical analysis, the system param-49 eters in these protocols are not designed to achieve the 50 optimal performance. In fact, analytical analysis is bene-51 ficial not only for better understanding the nature of a 52 proposed protocol, but also for better designing the system 53 parameters of a protocol to achieve the optimal perfor- 54 mance. It can also provide useful insights to guide the 55 future broadcast protocol designs in CR ad hoc networks. 56 Hence, in this paper, we focus on the analytical analysis of 57 broadcast protocols for multi-hop CR ad hoc networks. 58

Although a vast amount of analytical works on broadcast protocols in traditional MANETs exist [15]–[19], currently, there is no analytical work on broadcast protocols in multihop CR ad hoc networks. More importantly, all the methods proposed for traditional MANETs cannot be simply applied to multi-hop CR ad hoc networks. This is because that in traditional MANETs, the channel availability is uniform for

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Fig. 1. Single-hop broadcast scenario. (a) Traditional ad hoc networks. (b) CR ad hoc networks.

<sup>66</sup> all nodes, as shown in Fig. 1(a). However, in CR ad hoc
<sup>67</sup> networks, different secondary users (SUs) may acquire dif<sup>68</sup> ferent available channel sets, depending on the locations
<sup>69</sup> and traffic of primary users (PUs), as shown in Fig. 1(b).
<sup>70</sup> This non-uniform channel availability leads to several sig<sup>71</sup> nificant differences and causes unique challenges when
<sup>72</sup> analyzing the performance of broadcast protocols in CR ad
<sup>73</sup> hoc networks.

First of all, unlike in traditional MANETs, in CR ad 74 75 hoc networks, the single-hop broadcast is not always suc-76 cessful in an error-free environment. The reason can be <sup>77</sup> illustrated using Fig. 1. If node A is the source node, in tra-78 ditional MANETs, all its neighboring nodes can tune to the <sup>79</sup> same channel to receive the broadcast message. However, 80 in CR ad hoc networks, such a common available chan-<sup>81</sup> nel for all neighboring nodes may not exist [20]–[24]. As 82 a result, the broadcast may fail. More severely, even if a 83 common available channel exists between the source node 84 and its neighboring nodes, they may not be able to tune 85 to that channel at the same time, which will also result in 86 a failed broadcast. In fact, whether the single-hop broad-87 cast is successful depends on the channel availability of 88 each SU which is time-varying and location-varying. Due <sup>89</sup> to the uncertainty of the single-hop broadcast success, the 90 successful broadcast ratio of a network is usually random. 91 Furthermore, since there usually exist multiple message <sup>92</sup> propagation scenarios for all the nodes to successfully 93 receive the broadcast message in a multi-hop CR ad hoc net-<sup>94</sup> work, it is extremely challenging to identify every possible 95 message propagation scenario for calculating the success-96 ful broadcast ratio in a complicated network. An example <sup>97</sup> illustrating this challenge will be given in Section 2.1.

Secondly, different from traditional MANETs where the <sup>99</sup> relative locations of the communication pair do not impact 100 the successful receipt of the message as long as they are 101 within the transmission range of each other, in CR ad hoc 102 networks, the probability that a node successfully receives 103 a broadcast message is affected by the relative locations 104 between the sender and the receiver. This is because that 105 the available channels of a SU are obtained based on the 106 sensing outcome from the proximity of the node. Thus, SU 107 nodes that are close to each other have similar available 108 channels and they may have higher successful broadcast 109 ratio, as compared with the SU nodes far away from each 110 other whose available channels are often less similar. These 111 two differences show that the successful broadcast ratio is 112 affected by various factors and it is random. Currently, there <sup>113</sup> is no straightforward solution to analyze this issue.

Thirdly, the single-hop broadcast delay is usually more than one time slot in CR ad hoc networks, while in traditional MANETS, it is always one time slot. As shown in Fig. 1(a), 116 node A only needs one time slot to let all its neighbor- 117 ing nodes receive the broadcast message in an error-free 118 environment. However, in CR ad hoc networks, due to the 119 non-uniform channel availability, node A may have to use 120 multiple channels for broadcasting and may not be able 121 to finish the broadcast within one time slot. In fact, the 122 exact broadcast delay for all single-hop neighboring nodes 123 to successfully receive the broadcast message in CR ad hoc 124 networks relies on various factors (e.g., channel availability 125 and the number of neighboring nodes) and it is also random. 126 Moreover, since there may exist multiple message propagation scenarios, to identify which node is the last node in a 128 network to receive the message is very difficult. Thus, the 129 multi-hop broadcast delay is extremely difficult to obtain. 130

Finally, broadcast collisions are complicated in CR ad 131 hoc networks. Unlike in traditional MANETs where nodes 132 use a common channel for broadcasting, in CR ad hoc networks, nodes may use multiple channels for broadcasting. 134 Without the information about the channel used for broadcasting and the exact delay for a single-hop broadcast, to predict when and on which channel a broadcast collision 137 occurs is extremely difficult. Hence, to mathematically analyze broadcast collisions is very challenging for multi-hop CR ad hoc networks under practical scenarios. 140

In summary, due to the randomness of the single-hop 141 successful broadcast ratio and broadcast delay, the broad- 142 cast performance of a multi-hop CR ad hoc network is 143 extremely challenging to analyze. Currently, no existing 144 work on CR ad hoc networks addresses these challenges. 145 Moreover, due to the above explained differences, the ana- 146 lytical methodology for broadcast protocol analysis in tra- 147 dition MANETs cannot be extended to CR ad hoc networks. 148 Specifically, the existing performance analytical papers on 149 broadcasting in traditional multi-channel ad hoc networks 150 cannot reflect the unique features (e.g., non-uniform chan- 151 nel availability and channel rendezvous schemes) in multi- 152 hop CR ad hoc networks because: 1) a common control 153 channel is used for broadcasting [25]-[29]; 2) only single- 154 hop scenario is considered [25],[27],[30]; 3) a centralized 155 entity is needed to schedule the broadcast [30]; and 4) mul- 156 tiple radios are used [31]. Therefore, in this paper, we study 157 the performance analysis of broadcast protocols for multi- 158 hop CR ad hoc networks. A novel unified analytical model 159 is proposed to analyze the broadcast protocols in CR ad 160 hoc networks with any topology. Specifically, in this paper, 161 we propose to decompose an intricate network into sev- 162 eral simple networks which are tractable for analysis. We 163 also propose systematic methodologies for such decom- 164 position. The main contributions of this paper are given 165 as follows:

- An algorithm for calculating the successful broadcast 167 ratio (i.e., the probability that all nodes in a network successfully receive a broadcast message) is 169 proposed for CR ad hoc networks. The proposed 170 algorithm is a general methodology that can be 171 applied to any broadcast protocol proposed for 172 multi-hop CR ad hoc networks with any topology. 173
- 2) An algorithm for calculating the average broadcast delay 174 (i.e., the average duration from the moment a 175

broadcast starts to the moment the last node in 176 the network receives the broadcast message) is pro-177 posed for CR ad hoc networks under grid topology. 178 3) The derivation methods of the single-hop performance 179 metrics, successful broadcast ratio, average broad-180 cast delay, and broadcast collision rate (i.e., the 181 probability that a single-hop broadcast fails due to 182 broadcast collisions), for three different broadcast 183 protocols in CR ad hoc networks under practical scenarios (e.g., no dedicated common control channel 185 exists and the channel information of any other SUs 186 is not known) are proposed. 187

A hardware system is developed to implement different
 broadcast protocols in multi-hop CR ad hoc networks
 and validate our proposed unified analytical model.

<sup>191</sup> To the best of our knowledge, this is the first analytical <sup>192</sup> work on the performance analysis of broadcast protocols <sup>193</sup> for multi-hop CR ad hoc networks.

The rest of this paper is organized as follows. The algorithm for calculating the successful broadcast ratio proposed in Section 2. The proposed algorithm for approximating the average broadcast delay is presented in Section 3. In Section 4, three existing broadcast protocols for multi-hop CR ad hoc networks under practical scenarios and the derivations of their single-hop performance metrics are introduced. The proposed algorithms are validated in Section 5, followed by the conclusions in Section 6.

#### 203 2 CALCULATING THE SUCCESSFUL 204 BROADCAST RATIO

<sup>205</sup> In this section, we present the proposed algorithm for calcu-<sup>206</sup> lating the successful broadcast ratio of a broadcast protocol <sup>207</sup> in multi-hop CR ad hoc networks. We first introduce a <sup>208</sup> unique challenge of calculating the successful broadcast <sup>209</sup> ratio. Then, the details of the proposed algorithm are pre-<sup>210</sup> sented. In addition, an example is given to show the process <sup>211</sup> of the proposed algorithm. For simplicity, we assume that <sup>212</sup> the wireless channels are error-free (i.e., the white noise <sup>213</sup> of the channels is ignored). However, the probability that a <sup>214</sup> broadcast fails due to the channel noise can be easily added <sup>215</sup> in our analysis, if necessary. In the rest of the paper, we use <sup>216</sup> the term "sender" to indicate a SU who has just received <sup>217</sup> a broadcast message and will rebroadcast the message. In <sup>218</sup> addition, we use the term "receiver" to indicate a SU who <sup>219</sup> has not received the broadcast message yet.

#### 220 2.1 The Unique Challenge

<sup>221</sup> Let G(V, E) denote the topology of a CR ad hoc network, <sup>222</sup> where V is the set of all SU nodes in the network and E is <sup>223</sup> the set of all links in the network. The problem of calculat-<sup>224</sup> ing the successful broadcast ratio is described as: given a <sup>225</sup> CR ad hoc network G(V, E), from the source node  $v_s$ , every <sup>226</sup> other node follows a certain rule to rebroadcast (e.g., simple <sup>227</sup> flooding or the broadcast scheduling algorithm used in the <sup>228</sup> distributed broadcast scheme in [14]), what is the successful <sup>229</sup> broadcast ratio of G(V, E)?

As mentioned in Section 1, the single-hop successful broadcast ratio may not always be one in CR ad hoc networks due to various reasons. Therefore, a SU may not be able to receive the broadcast message from its direct



Fig. 2. Example for showing the unique challenge when calculating the successful broadcast ratio. (a)  $2 \times 2$  grid network. (b)  $2 \times 3$  grid network.

parent node. However, during the broadcast procedure, it <sup>234</sup> may receive the message from other nodes via different <sup>235</sup> paths in the network. This is different from the broad- <sup>236</sup> cast schemes in traditional MANETs, where nodes usually <sup>237</sup> receive broadcast messages from their parent nodes. This <sup>238</sup> feature imposes a special challenge of calculating the successful broadcast ratio for the whole CR ad hoc network. <sup>240</sup> That is, there exist multiple message propagation scenarios for all the nodes to successfully receive the message. <sup>242</sup> The overall successful broadcast ratio is the sum of the <sup>243</sup> successful broadcast ratio of all these propagation scenarios. However, it is extremely challenging to calculate the <sup>245</sup> successful broadcast ratio for every message propagation <sup>246</sup> scenario when the network topology is complicated. <sup>247</sup>

To further illustrate this challenge, we consider a sim- 248 ple 2  $\times$  2 grid network shown in Fig. 2(a), where node A <sup>249</sup> is the source node. There are four links in the network, 250 where the successful broadcast ratio over each link is given. 251 The single-hop successful broadcast ratio depends on the 252 specific broadcast protocol used. The method to obtain the 253 single-hop successful broadcast ratio may be different for 254 different protocols. We will explain the methods for calcu- 255 lating the single-hop successful broadcast ratio for various 256 protocols in Section 4. If simple flooding is used to propa- 257 gate the message, there are totally seven different scenarios 258 for all nodes to successfully receive the message. They are: 259 1)  $A \rightarrow B \rightarrow D \rightarrow C$ ; 2)  $A \rightarrow B \rightarrow D$  and  $A \rightarrow C$ ; 3)  $A \rightarrow B_{260}$ and  $A \to C \to D$ ; 4)  $A \to C \to D \to B$ ; 5)  $A \to B \to D$ , 261  $A \rightarrow C \rightarrow D$  and B, C do not have a collision at D; 6) 262  $A \rightarrow C \rightarrow D \rightarrow B$ ,  $A \rightarrow B$  and A, D do not have a colli- 263 sion at B; and 7)  $A \rightarrow B \rightarrow D \rightarrow C$ ,  $A \rightarrow C$  and A, D do 264 not have a collision at C. Accordingly, since the broadcast 265 events to different SU nodes are independent, the successful 266 broadcast ratio for these seven scenarios is:  $p_1(1-p_2)p_3p_4$ , 267  $p_1p_2p_3(1-p_4), p_1p_2(1-p_3)p_4, (1-p_1)p_2p_3p_4, p_1p_2p_3p_4-p_{q1}, 268$  $p_1p_2p_3p_4 - p_{q2}$ , and  $p_1p_2p_3p_4 - p_{q2}$ , where  $p_{q1}$  is the proba- 269 bility that B and C fail to broadcast to D due to broadcast 270 collisions and  $p_{q2}$  is the probability that A and D fail to 271 broadcast due to broadcast collisions. The probability that 272 two nodes have a collision also depends on the specific 273 broadcast protocol used. Therefore, the overall successful 274 broadcast ratio is the sum of the successful broadcast ratio 275 of these seven scenarios, that is, 276

$$P_{succ} = p_1(1-p_2)p_3p_4 + p_1p_2p_3(1-p_4) + p_1p_2(1-p_3)p_4 + (1-p_1)p_2p_3p_4 + (p_1p_2p_3p_4 - p_{a_1}) + 2(p_1p_2p_3p_4 - p_{a_2}).$$
(1) 277

Then, we increase the dimension of the grid network to  $_{278}$  2 × 3, as shown in Fig. 2(b). If simple flooding is used, the  $_{279}$  total number of message propagation scenarios is 40. The  $_{280}$ 

TABLE 1 Notations Used in the Proposed Algorithm 1

E(v)	The set of all the links that connect to node $v$
e(v, u)	The link that connects node $v$ and $u$
P(v, u)	The successful broadcast ratio from node $v$ to $u$
P(G(V, E))	The successful broadcast ratio of the network $G(V, E)$
$P_q(v, u, k)$	The probability that node $v$ and $u$ fail to broadcast to node $k$ due to broadcast collisions
·	The number of elements in a set

<sup>281</sup> overall successful broadcast ratio is the sum of the suc-<sup>282</sup> cessful broadcast ratio of all these 40 message propagation <sup>283</sup> scenarios. Note that although only 2 additional nodes and 3 <sup>284</sup> additional links are added, the total number of propagation <sup>285</sup> scenarios increases significantly. Moreover, if the grid net-<sup>286</sup> work size is  $2 \times 4$ , the total number of message propagation <sup>287</sup> scenarios is 252. If we further increase the dimension of the <sup>288</sup> grid network to  $3 \times 3$ , it is almost impossible to obtain the <sup>289</sup> successful broadcast ratio of every possible message propa-<sup>290</sup> gation scenario. Therefore, when the number of nodes and <sup>291</sup> links increases in a CR ad hoc network, the total number <sup>292</sup> of message propagation scenarios increases exponentially. It <sup>293</sup> is extremely challenging to identify every possible message <sup>294</sup> propagation scenario and calculate the successful broadcast <sup>295</sup> ratio for each scenario in a complicated network.

#### 296 2.2 The Proposed Algorithm

<sup>297</sup> We develop an iterative algorithm to address the above <sup>298</sup> challenge. The main idea of the proposed algorithm is to <sup>299</sup> decompose a complicated network into a few simpler net-<sup>300</sup> works so that the successful broadcast ratio of these simpler <sup>301</sup> networks is straightforward to obtain and the complexity <sup>302</sup> of the original network can be reduced. Then, the success-<sup>303</sup> ful broadcast ratio of the overall network can be acquired. <sup>304</sup> The notations used in the proposed algorithm are listed <sup>305</sup> in Table 1. The pseudo-codes of the proposed algorithm <sup>306</sup> for calculating the successful broadcast ratio is shown in <sup>307</sup> Algorithm 1.

Under the proposed algorithm, at each iteration round, a link that connects to the source node is randomly selected. Based on whether the broadcast over this link is successrul ful or not, the network is decomposed into two simpler networks. If the broadcast over this link is successful, all links that connect to the other node of the selected link will connect to the source node. If the broadcast over this link fails, this link is simply removed from the network. The successful broadcast ratio over each remaining link is updated accordingly after each iteration. The process terminates when only two nodes are left in the remaining networks.

#### 320 2.3 An Illustrative Example

<sup>321</sup> We use an example to illustrate the process of the pro-<sup>322</sup> posed Algorithm 1. As shown in Fig. 3(a), the original CR <sup>323</sup> ad hoc network consists of 4 nodes and 5 links. Based on <sup>324</sup> Algorithm 1, since the source node  $v_s$  has two links, we <sup>325</sup> randomly select one of these two links (e.g., link  $e(v_s, v_2)$ ). <sup>326</sup> In the first iteration, if the broadcast over the link  $e(v_s, v_2)$ <sup>327</sup> is successful, all nodes that are originally connected to  $v_2$ <sup>328</sup> are connected to the source node, as shown in Fig. 3(b). <sup>329</sup> In addition, the successful broadcast ratios of the new



Fig. 3. Process of the proposed Algorithm 1 for a 4-node CR ad hoc network. (a) original network. (b) Link  $e(v_s, v_2)$  is successful. (c) Link  $e(v_s, v_2)$  is failed. (d) Link  $e(v_s, v_1)$  is successful after (b). (e) Link  $e(v_s, v_1)$  is failed after (b). (f) Link  $e(v_s, v_1)$  is successful after ??.

**Algorithm 1:** The proposed algorithm for calculating the successful broadcast ratio.

```
Input: The topology of the network G(V, E), the source node v_s.
Output: P(G(V, E)).
if |V| > 2 then
    if |E(v_s)| > 1 then
         E_1 \leftarrow E; V_1 \leftarrow V;
                                              /* initialization */
         E_2 \leftarrow E; V_2 \leftarrow V;
         Randomly select e(v_s, v_i) \in E(v_s);
         foreach v_k, e(v_i, v_k) \in E(v_i) do
                                            /* original link to v_i
              E_1 \leftarrow E_1 + e(v_s, v_k);
              is connected to v_s */
              if e(v_s, v_k) \in E(v_s) then
                   P(v_s, v_k) \leftarrow
                   1 - (1 - P(v_i, v_k))(1 - P(v_s, v_k)) - P_q(v_s, v_i, v_k);
                   /* update the link success ratio */
              else
                P(v_s, v_k) \leftarrow P(v_i, v_k);
         E_1 \leftarrow E_1 - E(v_i);
                                 /* remove all links to v_i * /
         V_1 \leftarrow V_1 - v_i;
                                                       /* remove v_i * /
         E_2 \leftarrow E_2 - e(v_s, v_i);
                                                     remove e(v_s, v_i) * /
         P(G(V, E)) \leftarrow
         P(v_s, v_i)P(G_1(V_1, E_1)) + (1 - P(v_s, v_i))P(G_2(V_2, E_2));
         /* calculate the successful ratio from the
         two simpler networks */
         return P(G(V, E));
     else if |E(v_s)| = 1 then
         E_1 \leftarrow E; V_1 \leftarrow V;
         select e(v_s, v_i) \in E(v_s);
         foreach v_k, e(v_i, v_k) \in E(v_i) do
              E_1 \leftarrow E_1 + e(v_s, v_k);
              P(v_s, v_k) \leftarrow P(v_i, v_k);
         E_1 \leftarrow E_1 - E(v_i);
          V_1 \leftarrow V_1 - v_i;
         P(G(V, \vec{E})) \leftarrow P(v_s, v_i)P(G_1(V_1, E_1));
         return P(G(V, E));
else if |V| = 2 then
    select e(v_s, v_i) \in E(v_s);
    return P(v_s, v_i);
                                    /* iteration terminates */
```

links are updated. That is,  $P(v_s, v_3) = P(v_2, v_3) = p_5$  and  ${}^{330}p'_1 = 1 - (1 - p_1)(1 - p_3) - P_q(v_s, v_2, v_1)$  because the mes-  ${}^{331}$  sage propagation scenarios in the original network for  $v_1$   ${}^{332}$  to successfully receive the message directly from  $v_s$  or  ${}^{333}$ 



Fig. 4. Example for showing the randomness of the single-hop broadcast delay in CR ad hoc networks. (a) B is on channel 1. (b) B is on channel 5.

 $v_2$  are: 1)  $v_s \rightarrow v_1$  only; 2)  $v_s \rightarrow v_2 \rightarrow v_1$  only; and 3) 335  $v_s 
ightarrow v_1, v_s 
ightarrow v_2 
ightarrow v_1$  and  $v_s, v_2$  do not have a collision 336 at  $v_1$ . The probability  $(1-p_1)(1-p_3)$  in calculating  $p'_1$  is the 337 probability that both  $v_s$  and  $v_2$  fail to broadcast to  $v_1$ . In <sup>338</sup> addition, the probability that node  $v_s$  and  $v_2$  fail to broad-<sup>339</sup> cast to node  $v_1$  due to broadcast collisions  $P_q(v_s, v_2, v_1)$  will 340 be calculated in Section 4. On the other hand, if the broad-<sup>341</sup> cast over the link  $e(v_s, v_2)$  fails, this link is simply removed 342 from the network, as shown in Fig. 3(c). The successful 343 broadcast ratio of the original network can be obtained 344 from the successful broadcast ratio of the two simpler net-<sup>345</sup> works, as shown in Fig. 3(b) and (c). In the second iteration, 346 these two simpler networks can be further decomposed 347 following the same procedure. For the network shown in <sup>348</sup> Fig. 3(b), assume that we select the link  $e(v_s, v_1)$ . Similar 349 to the process of the first iteration, this network is further 350 decomposed into two networks, as shown in Fig. 3(d) and 351 (e), where  $p'_5 = 1 - (1 - p_4)(1 - p_5) - P_a(v_s, v_1, v_3)$ . Then, the 352 successful broadcast ratio of the network shown in Fig. 3(b) 353 can be obtained from the successful broadcast ratio of these 354 two new networks shown in Fig. 3(d) and (e). For the net-355 work shown in Fig. 3(c), since the source node has only 356 one link, this link must be successful for other nodes to 357 receive the message. Thus, this network is reduced to the 358 network shown in Fig. 3(f) and the successful broadcast 359 ratio of this network can be obtained from the successful 360 ratio of the network shown in Fig. 3(f). Therefore, if we <sup>361</sup> repeat this process, the complexity of the networks from the <sup>362</sup> second iteration can be further reduced. Finally, the original 363 network can be decomposed into several single-hop net-<sup>364</sup> works. Then, the procedure of the proposed Algorithm 1 365 terminates. Therefore, the successful broadcast ratio of the 366 original network can be expressed as

$$P_{succ} = p_{2}\{[1 - (1 - p_{1})(1 - p_{3}) - P_{q}(v_{s}, v_{2}, v_{1})][1 - (1 - p_{4}) \\ (1 - p_{5}) - P_{q}(v_{s}, v_{1}, v_{3})] + [(1 - p_{1})(1 - p_{3}) + P_{q}(v_{s}, v_{2}, v_{1})]p_{4}p_{5}\}$$
(2)  
+  $(1 - p_{2})p_{1}\{p_{3}[1 - (1 - p_{4})(1 - p_{5}) - P_{q}(v_{s}, v_{2}, v_{3})] + (1 - p_{3})p_{4}p_{5}\}.$ 

#### 368 3 CALCULATING THE AVERAGE BROADCAST 369 DELAY

<sup>370</sup> In this section, we introduce the proposed algorithm for <sup>371</sup> calculating the average broadcast delay of a broadcast pro-<sup>372</sup> tocol. Similar to the previous section, we first present the <sup>373</sup> unique challenge of calculating the average broadcast delay <sup>374</sup> for a CR ad hoc network. Then, the detailed algorithm is <sup>375</sup> given. Furthermore, an example is shown to illustrate the <sup>376</sup> process of the proposed algorithm.

#### 377 3.1 The Unique Challenge

<sup>378</sup> As mentioned in Section 1, since the single-hop broadcast <sup>379</sup> delay depends on various factors, such as the channel avail-<sup>380</sup> ability of the communication pair and specific broadcast

Fig. 5. Example of a 8-node CR ad hoc network with the levels of SUs.

1=2

I = 1

protocol, the single-hop broadcast delay is random. Fig. 4 381 illustrates the randomness of the single-hop broadcast delay 382 in CR ad hoc networks. In Fig. 4, node A is the sender and 383 broadcasts the message on each available channel sequen- 384 tially. In addition, node B is the receiver and constantly 385 listens on the channel shown in the bold font. Since node 386 B does not have any information about the sender before 387 a broadcast starts, the channel it stays on is randomly 388 selected. It is shown that, even though the channel avail- 389 ability of node B is the same in the two scenarios shown 390 in Fig. 4(a) and (b), the single-hop broadcast delay is quite 391 different (i.e., it takes 1 time slot for a successful broad- 392 cast in Fig. 4(a), while it takes 5 time slots for a successful 393 broadcast in Fig. 4(b)). Hence, due to this randomness, to 394 obtain the single-hop broadcast delay in CR ad hoc net- 395 works is challenging. Moreover, if the number of senders 396 and receivers is larger than one, it is even more difficult. 397

#### 3.2 The Proposed Algorithm

G

1=2

Since to obtain the closed form expression of the average 399 broadcast delay for arbitrary network topology is extremely 400 complicated, in this paper, we focus on the grid topology. 401 However, the proposed methodology can be applied to any 402 network topology. We define the level of SUs as h if they 403 are h hops to the source node (denoted as L = h). Fig. 5 404 shows an example of an 8-node CR ad hoc network with 405 the levels of SUs where A is the source node. Then, the 406 original network is decomposed into  $H_m$  levels, where  $H_m$  407 is the distance from the source node to the furthest node 408 in the network. To make the derivation process tractable, 409 we first make two assumptions. First of all, we assume 410 that the broadcast message is propagated from the source 411 node to the furthest node sequentially based on the relative 412 distance to the source node. This means that, we assume 413 that the nodes who are closer to the source node receive 414 the message sooner than the nodes who are farther away 415 from the source node. Based on this assumption, we cat- 416 egorize the SUs based on their relative distances to the 417 source node. We further justify this assumption using sim- 418 ulation. We apply the broadcast protocol proposed in [13] 419 to the network shown in Fig. 5. Fig. 6 shows the simulation 420 results of the average delay for different nodes to receive 421 the broadcast message in the network shown in Fig. 5. It 422 is shown that nodes at a higher level (e.g., nodes D and 423 *E* at the second level) receive the broadcast message later 424 than the nodes at a lower level on average (e.g., nodes  $B_{425}$ and C at the first level), which justifies our first assump- 426 tion. The second assumption is that only the nodes that are 427 at the highest level or have a path leading to the furthest 428 node (excluding the source node) contribute to the overall 429 average broadcast delay. Other nodes will be removed from 430 the network for calculating the average broadcast delay. 431



Fig. 6. Average delay for different nodes to receive the broadcast message in the network shown in Fig. 5.

<sup>432</sup> This assumption is straightforward since those nodes are <sup>433</sup> independent of the message propagation path to the nodes <sup>434</sup> at the highest level. For instance, in Fig. 5, nodes *G* and *H* <sup>435</sup> do not contribute to the message propagation to node *F*. <sup>436</sup> Thus, they can be removed when calculating the average <sup>437</sup> broadcast delay of the network.

<sup>438</sup> The main idea of the proposed algorithm is that the <sup>439</sup> overall average broadcast delay is the sum of the average <sup>440</sup> broadcast delay at each level. At each level, it is a simple <sup>441</sup> network whose average broadcast delay can be obtained. <sup>442</sup> That is,  $\Gamma = \sum_{i}^{H_m} D_i$ , where  $\Gamma$  is the overall average broad-<sup>443</sup> cast delay and  $D_i$  is the average broadcast delay of the <sup>444</sup> nodes at level *i*.

Then, we calculate the average broadcast delay at level 445  $_{446}$  *i*,  $D_i$ . Based on the number of parent nodes, there exist only 447 two scenarios of the single-hop broadcast in a grid topol-448 ogy network. The first scenario is that a SU only has one 449 parent node (denoted as Scenario I, as shown in Fig. 7(a)), 450 while the second scenario is that a SU has two parent nodes 451 (denoted as Scenario II, as shown in Fig. 7(b)). We further <sup>452</sup> prove that the maximum number of parent nodes for a node 453 in grid topology networks is two. The proof is: if there are 454 more than two parent nodes (say, three), these three nodes 455 should be at the same level. However, for any node that is 456 the parent node of any two of those parent nodes (exactly 457 1-hop away), it needs more than two hops to reach the 458 third parent node. That is, these three nodes cannot be at 459 the same level. Therefore, only the two single-hop broad-460 cast scenarios shown in Fig. 7 exist. We assume that for <sup>461</sup> the nodes at the same level, there are  $\alpha$  Scenario I and  $\beta$ 462 Scenario II.

If the current level, level *i*, is not the highest level, the average broadcast delay at level *i* is the mean of the singlehop average broadcast delay of the nodes at level *i*. That is,  $D_i = (\alpha \tau_1 + \beta \tau_2)/(\alpha + \beta)$ , where  $\tau_1$  and  $\tau_2$  are the single-hop average broadcast delay of Scenario I and II, respectively. Denote the probabilities that the single-hop broadcast is successful at time slot *k* as  $P_I(k)$  and  $P_{II}(k)$  for Scenario I and II, respectively.  $P_I(k)$  and  $P_{II}(k)$  can be obtained based on a specific broadcast protocol, which is explained in Section 4. Given a successful broadcast, we first obtain the conditional probability that the single-hop broadcast is successful at time slot *k* for the two scenarios:

75 
$$P_1(k) = \frac{P_I(k)}{\sum_j P_I(j)},$$
  
76  $P_2(k) = \frac{P_{II}(k)}{\sum_i P_{II}(j)}.$ 



Fig. 7. Two single-hop broadcast scenarios in a grid topology network. (a) Scenario I. (b) Scenario II.

Therefore, we have  $\tau_1 = \sum_{k=1}^{T_m} kP_1(k)$  and  $\tau_2 = \sum_{k=1}^{T_m} kP_2(k)$ , 477 where  $T_m$  is the maximum length of time slots the sender 478 uses for broadcasting. 479

If the current level is the highest level, the calculation  $^{490}$  method for  $D_i$  is different. Since the probability that the  $^{491}$  broadcast is successful at time slot k is different in the  $^{492}$  two broadcast scenarios, we need to consider two cases:  $^{483}$  the last SU node at level i successfully receives the broad-  $^{484}$  cast message is under Scenario I or Scenario II. Therefore,  $^{495}$  the broadcast message at time slot d is under Scenario  $^{477}$  I and no other SU receives the message at time slot d  $^{489}$  under Scenario II. Thus, we have the probability that the  $^{489}$  single-hop broadcast delay is d at level i as  $^{490}$ 

$$P'(D_i = d) = {\alpha \choose 1} P_1(d) \left[ \sum_{k=1}^d P_1(k) \right]^{\alpha - 1} \left[ \sum_{k=1}^{d-1} P_2(k) \right]^{\beta}.$$
 (4) 491

Next, we assume that the last SU node successfully receives 492 the broadcast message at time slot *d* under Scenario II and 493 no other SU node receives the message at time slot *d* under 494 Scenario I. Thus, we obtain 495

$$P''(D_i = d) = {\beta \choose 1} P_2(d) \left[ \sum_{k=1}^{d-1} P_1(k) \right]^{\alpha} \left[ \sum_{k=1}^{d} P_2(k) \right]^{\beta-1}.$$
 (5) 496

Last, we assume that under both scenarios, at least one  $^{497}$  node receives the broadcast message at time slot *d*. Hence,  $^{498}$  we have

$$P^{\prime\prime\prime}(D_{i}=d) = \binom{\alpha}{1} \binom{\beta}{1} P_{1}(d) P_{2}(d) \left[ \sum_{k=1}^{d-1} P_{1}(k) \right]^{\alpha-1} \left[ \sum_{k=1}^{d-1} P_{2}(k) \right]^{\beta-1}.$$
(6) 501

Therefore, the probability that the single-hop broadcast  $_{502}$  delay is *d* at level *i* can be written as  $_{503}$ 

$$\Pr(D_i = d) = P'(D_i = d) + P''(D_i = d) + P'''(D_i = d).$$
(7) 50

Then, the average broadcast delay at level *i* is

$$D_i = \sum_{d=1}^{T_m} d\Pr(D_i = d).$$
 (8) 506

505

507

#### 3.3 An Illustrative Example

(3)

We use the example shown in Fig. 5 to illustrate the 508 proposed algorithm for calculating the average broadcast 509 delay. From Fig. 5, there are three levels of nodes in the 510 network. As explained above, according to our second 511

Тх	1	3	4	1	5	4	3	4	5	3	5	2
Rx	3	1	2	4	3	2	1	2	4	3	2	1

Fig. 8. Example of the random broadcast scheme.

<sup>512</sup> assumption, we first remove nodes G and H for the consid-<sup>513</sup> eration of average broadcast delay. Then, at the first level, <sup>514</sup> since both nodes B and C are under Scenario I, for  $D_1$ , <sup>515</sup> we have

516 
$$D_1 = \tau_1 = \sum_{k=1}^{T_m} \frac{k P_I(k)}{\sum_j P_I(j)}.$$
 (9)

<sup>517</sup> That is, the average broadcast delay at level 1 is the same <sup>518</sup> as the single-hop broadcast delay under Scenario I. At the <sup>519</sup> second level, nodes D and E are under different scenarios. <sup>520</sup> Therefore, we have

<sup>521</sup> 
$$D_2 = \frac{\tau_1 + \tau_2}{2} = \frac{1}{2} \left[ \sum_{k=1}^{T_m} \frac{k P_I(k)}{\sum_j P_I(j)} + \sum_{k=1}^{T_m} \frac{k P_{II}(k)}{\sum_j P_{II}(j)} \right].$$
 (10)

<sup>522</sup> Finally, for  $D_3$ , since this is the highest level,  $D_3$  can be <sup>523</sup> obtained using (8), where  $\alpha = 0$  and  $\beta = 1$ . That is,

524 
$$D_3 = \sum_{d=1}^{T_m} d \frac{P_{II}(d)}{\sum_j P_{II}(j)}.$$
 (11)

<sup>525</sup> By summing up the average broadcast delay of these three <sup>526</sup> levels, the overall average broadcast delay for the network <sup>527</sup> shown in Fig. 5 can be written as  $\Gamma = \sum_{i=1}^{3} D_i$ .

#### 528 4 BROADCASTING IN CR AD HOC NETWORKS

<sup>529</sup> In this section, we first introduce several existing broad-<sup>530</sup> cast designs, i.e., the random scheme and the schemes <sup>531</sup> proposed in [13],[14], for CR ad hoc networks under <sup>532</sup> practical scenarios. Since the broadcast schemes proposed <sup>533</sup> in [11] and [12] are based on impractical assumptions <sup>534</sup> (i.e., a dedicated common control channel for the whole <sup>535</sup> network is employed and the available channel informa-<sup>536</sup> tion of all SUs are assumed to be known), we exclude <sup>537</sup> these proposals in this paper. In addition, we propose the <sup>538</sup> derivation methods to calculate the single-hop broadcast <sup>539</sup> performance metrics (i.e., successful broadcast ratio, aver-<sup>540</sup> age broadcast delay, and broadcast collision rate) for each <sup>541</sup> protocol.

#### 542 4.1 Random Broadcast Scheme

<sup>543</sup> The first broadcast scheme is called the *random broadcast* <sup>544</sup> *scheme*. Since a SU is unaware of the channel availability <sup>545</sup> information of other SUs before broadcasts are executed, <sup>546</sup> a straightforward action for a SU sender is to randomly <sup>547</sup> select a channel from its available channel set and broad-<sup>548</sup> casts a message on that channel in a time slot. If the channel <sup>549</sup> selected by the receiver is the same as the channel selected <sup>550</sup> by the sender, the broadcast message can be successfully <sup>551</sup> received. Fig. 8 illustrates the procedure of the random <sup>552</sup> broadcast scheme, where the shaded part represents a <sup>553</sup> successful broadcast.

#### 4.1.1 Single-Hop Successful Broadcast Ratio for the Random Broadcast Scheme

We first calculate the single-hop successful broadcast ratio 556 for the random broadcast scheme. Without loss of generality, in the rest of the paper, the sender and the receiver of 558 the single-hop link is denoted as A and B. We further denote 559 the numbers of available channels for the single-hop communication pair as  $N_A$  and  $N_B$ , respectively. The number of 561 common channels between A and B is  $Z_{AB}$ . Therefore, the 562 probability that the single-hop broadcast is successful in a 563 time slot is 564

$$p_r = \binom{Z_{AB}}{1} \frac{1}{N_A} \frac{1}{N_B} = \frac{Z_{AB}}{N_A N_B}.$$
 (12) 566

Therefore, if the length of the time slots that the sender uses 566 for broadcasting is  $S_r$ , the single-hop successful broadcast 567 ratio for the random broadcast scheme is 568

$$P_{rand} = 1 - \left(1 - \frac{Z_{AB}}{N_A N_B}\right)^{S_r}.$$
 (13) 569

#### 4.1.2 Single-Hop Average Broadcast Delay for the Random Broadcast Scheme

Next, we calculate the single-hop average broadcast delay 572 for the random broadcast scheme. In this paper, since we 573 focus on grid topology for the broadcast delay, we only 574 need to consider the two single-hop broadcast scenarios 575 shown in Fig. 7. For Scenario I, since the sender and the 576 receiver randomly select a channel in a time slot, the prob- 577 ability that the single-hop broadcast is successful at time 578 slot k is  $P_I(k) = (1 - p_r)^{k-1} p_r$ , where  $p_r$  is given in (12). 579 For scenario II, since there are two senders, we denote the 580 other sender as C and the number of available channels 581 of C is  $N_{\rm C}$ . In addition, the number of common channels 582 between B and C is  $Z_{BC}$ . Thus, similar to (12), the proba-583 bility that the single-hop broadcast is successful between C 584 and B in a time slot is  $p_m = \frac{Z_{BC}}{N_B N_C}$ . Hence, the probability 585 that the single-hop broadcast is successful under Scenario 586 II in a time slot is  $p_{r2} = [1 - (1 - p_r)(1 - p_m)] - p_{q1}$ , where 587  $p_{a1}$  is the probability that nodes A and C have a broad- 588 cast collision at node B in a time slot. The derivation of 589  $p_{q1}$  is given in Section 4.1.3. Hence, the probability that 590 the single-hop broadcast is successful at time slot k can be 591 expressed as 592

$$P_{II}(k) = (1 - p_{r2})^{k-1} p_{r2}.$$
 (14) 593

Then, based on (3), given the single-hop broadcast is <sup>594</sup> successful, the conditional probability that the receiver successfully receives the broadcast message at time slot k for <sup>596</sup> both scenarios under the random broadcast scheme,  $P_1(k)$  <sup>597</sup> and  $P_2(k)$ , can be obtained. <sup>598</sup>

#### 4.1.3 Single-Hop Broadcast Collision Rate for the Random Broadcast Scheme

Next, we calculate the single-hop broadcast collision rate  $_{601}$  for the random broadcast scheme. We first derive the prob- $_{602}$  ability that nodes *A* and *C* have a broadcast collision  $_{603}$  at node *B* in a time slot,  $p_{q1}$ .  $p_{q1}$  is equivalent to the

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	◄ S											
Тх	3	6	3	6	3	6	3	6	3	6	3	6
Rx	1	1	2	2	6	6	1	1	2	2	6	6
	-		<u> </u>	× mi		•						

Fig. 9. Example of the QoS-based broadcast scheme.

<sup>604</sup> probability that all the three nodes select the same channel. <sup>605</sup> Denote the number of common channels among the three <sup>606</sup> nodes as  $Z_{ABC}$ . Thus, we have

607 
$$p_{q1} = \frac{Z_{ABC}}{N_A N_B N_C}.$$
 (15)

Since the length of the time slots that the sender uses for broadcasting is  $S_r$ , the probability that a single-hop broadcast fails due to broadcast collisions for the random broadcast scheme can be written as

612 
$$P_q(A, C, B) = \sum_{l=1}^{S_r} {S_r \choose l} p_{q1}^l \left[ (1-p_r)(1-p_m) \right]^{S_r-l}, \quad (16)$$

<sup>613</sup> where l is the number of time slots when nodes A and C<sup>614</sup> have a broadcast collision at node B.

#### 615 4.2 QoS-Based Broadcast Scheme

616 The second scheme is called the QoS-based broadcast 617 scheme [13], [32]. The main idea of the QoS-based broadcast 618 scheme is to let the sender broadcast on a subset of its 619 available channels in order to reduce the broadcast delay. 620 In addition, the channel hopping sequences of both the 621 sender and the receiver are designed for guaranteed ren-622 dezvous, given that the sender and the receiver have at least 623 one channel in common in their hopping sequences. Fig. 9 624 shows an example of the QoS-based broadcast scheme. For 625 each sender, it randomly selects n channels from its avail-626 able channel set. Then, it hops and broadcasts periodically  $_{627}$  on the selected *n* channels for *S* time slots. The values of  $_{628}$  n and S are determined by the QoS requirements of the 629 network (i.e., the successful broadcast ratio and the aver-630 age broadcast delay). On the other hand, for each receiver, 631 it first forms a random sequence that consists of its every  $_{632}$  available channel with a length of *n* time slots for each 633 channel. Then, it hops and listens following this sequence 634 periodically.

635

636

656

#### 4.2.1 Single-Hop Successful Broadcast Ratio for the QoS-Based Broadcast Scheme

We continue to use the notations for calculating the singlehop performance metrics in the random broadcast scheme 638 for the QoS-based broadcast scheme. Denote the number 639 of channels in the *n* channels selected by node *A* which 640 are also in the available channel set of node *B* as *y*. We 641 assume that the length of time slots that the sender uses 642 for broadcasting, *S*, is a multiple of *n*. Thus, the singlehop successful broadcast ratio for the QoS-based broadcast protocol is 645

$$P_{qos} = \sum_{y=y^*}^{y} H(y), \tag{17}$$
 646

where  $y^* = \max(1, n + Z_{AB} - N_A)$ ,  $y^{**} = \min(n, Z_{AB})$ , and 647 H(y) is written as 648

$$H(y) = \begin{cases} \frac{\binom{Z_{AB}}{y}\binom{N_A - Z_{AB}}{n - y}}{\binom{N_B}{n}} \frac{\binom{N_B}{y} - \binom{N_B - \frac{S}{n}}{y}}{\binom{N_B}{y}}, & \text{if } y < N_B - \frac{S}{n} \\ \frac{\binom{Z_{AB}}{y}\binom{N_A - Z_{AB}}{n - y}}{\binom{N_A - Z_{AB}}{n}}, & \text{if } y \ge N_B - \frac{S}{n}, \end{cases}$$
(18) 649

where  $\frac{\binom{Z_{AB}}{n-y}\binom{N_A-Z_{AB}}{n-y}}{\binom{N_A}{n}}$  is the probability that there are *y* com- <sup>650</sup> mon channels between the sender and the receiver in the <sup>651</sup> selected *n* channels by the sender. (18) indicates that when <sup>652</sup>

# *S* is large enough (the case when $y \ge N_B - \frac{S}{n}$ ), the single-hop successful broadcast ratio is independent of *S*.

#### 4.2.2 Single-Hop Average Broadcast Delay for the QoS-Based Broadcast Scheme

Secondly, we calculate the single-hop average broadcast <sup>657</sup> delay for the QoS-based broadcast scheme. Similar to the <sup>658</sup> random broadcast scheme, we first calculate the probabil- $^{659}$ ity that the single-hop broadcast is successful at time slot <sup>660</sup> k. Based on the broadcast protocol shown in Fig. 9, one <sup>661</sup> cycle of the broadcasting sequence of the receiver consists <sup>662</sup> of  $N_B$  sections, where each section includes the same channel repeated for n times. If the channel in a section is the <sup>664</sup> first appearing common available channel of nodes A and <sup>665</sup> B, the single-hop broadcast is successful within that section. Denote the sections of one cycle of the broadcasting <sup>667</sup> sequence of the receiver as  $[f_1, f_2, \ldots, f_{N_B}]$ . We calculate the <sup>668</sup> probability that for a particular y, the channel in  $f_i$  is the first <sup>669</sup> appearing common available channel,  $\Pr(f_i), i \in [1, N_B-y+1]$ . <sup>670</sup> This probability is equal to the probability that the first <sup>671</sup>

$$P_{II}(k) = \begin{cases} \sum_{y=y^*}^{y^{**}} \frac{\binom{Z_{AB}}{y}\binom{N_A-Z_{AB}}{n-y}}{\binom{N_A}{n}} \frac{\binom{N_B-\lfloor\frac{k-1}{n}\rfloor-1}{y-1}}{n\binom{N_B}{y}}, & \text{if } k \le n(N_B-y) \\ \sum_{y=y^*}^{y^{**}} \frac{\binom{Z_{AB}}{y}\binom{N_A-Z_{AB}}{n-y}}{\binom{N_A}{n}} \frac{1}{n\binom{N_B}{y}}, & \text{if } n(N_B-y) < k \le n(N_B-y+1) \\ 0, & \text{if } k > n(N_B-y+1). \end{cases}$$

$$P_{II}(k) = \begin{cases} \sum_{y=y^*}^{y^{**}} \sum_{x=x^*}^{q^{**}} \sum_{q=0}^{q^{**}} \frac{\binom{Z_{AB}}{y}\binom{N_A-Z_{AB}}{n-y}}{\binom{N_A}{n}} \frac{\binom{N_B-\lfloor\frac{k-1}{n}\rfloor-1}{n\binom{N_B}{y-2}-q-1}}{n\binom{N_B}{2y-2q-1}} Pr(x) Pr(q), & \text{if } k \le n(N_B-2y+2q) \\ \sum_{y=y^*}^{y^{**}} \sum_{x=x^*}^{q^{**}} \sum_{q=0}^{q^{**}} \frac{\binom{Z_{AB}}{y}\binom{N_A-Z_{AB}}{n-y}}{\binom{N_A}{n}} \frac{1}{n\binom{N_B}{2y-2q-1}} Pr(x) Pr(q), & \text{if } n(N_B-2y+2q) < k \le n(N_B-2y+2q+1) \\ 0, & \text{if } k > n(N_B-2y+2q+1). \end{cases}$$

$$(20)$$

<sup>672</sup> ball is in the *i*-th box if *y* balls are randomly put in  $N_B$ <sup>673</sup> boxes. Therefore,  $\Pr(f_i) = \frac{\binom{N_B-i}{y-1}}{\binom{N_B}{y}}$ . Since time slot *k* is in <sup>674</sup> the  $(\lfloor \frac{k-1}{n} \rfloor + 1)$ -th section, the probability that the single-<sup>675</sup> hop broadcast is successful in  $f_{\lfloor \frac{k-1}{n} \rfloor + 1}$  is  $\frac{\binom{N_B-\lfloor \frac{k-1}{n} \rfloor}{\binom{N_B}{y}}$ . On <sup>676</sup> the other hand, given that the first appearing common <sup>677</sup> available channel is in  $f_{\lfloor \frac{k-1}{n} \rfloor + 1}$ , since the channels in the <sup>678</sup> broadcasting sequence of the sender is evenly distributed, <sup>679</sup> the conditional probability that the broadcast is successful <sup>660</sup> in time slot *k* is  $\frac{1}{n}$ . Therefore, for Scenario I, the probability <sup>661</sup> that the single-hop broadcast is successful at time slot *k* is <sup>662</sup> expressed in (19).

For Scenario II, for simplicity, we assume that both the 683 684 two senders have the same number of common available 685 channels with the receiver (i.e.,  $Z_{AB} = Z_{BC}$ ). In addition, 686 the numbers of channels that are also available for the  $_{687}$  receiver in the selected *n* channels by the two senders are the same (denoted as y). Denote the number of chan-689 nels in the available channel sets of the two senders that  $_{690}$  are also available for all three nodes as x. Therefore, the <sup>691</sup> probability that there are x channels that are available for 692 all three nodes in their selected available channel sets is <sup>692</sup> Pr(x) =  $\left(\frac{Z_{ABC}}{Z_{AB}}\right)^x \left(1 - \frac{Z_{ABC}}{Z_{AB}}\right)^{y-x}$ , where  $Z_{ABC}$  is the number <sup>693</sup> of channels that are available for all three nodes. Therefore, 695 the probability that the single-hop broadcast is success-696 ful at time slot k under Scenario II is written in (20), 697 where Pr(q) is the probability that there are q channels out  $_{698}$  of x channels appearing in the same time slots. In addi-699 tion,  $x^* = \max(0, y - Z_{AB} + Z_{ABC}), x^{**} = \min(y, Z_{ABC}),$  and 700  $q^* = \min(x, y - 1)$ . Thus,  $\Pr(q)$  is written as 

701 
$$\Pr(q) = \begin{cases} \frac{\binom{x}{q} [(n-q)! - \sum_{j=1}^{x-q} (-1)^{(j+1)} \binom{x-q}{j} (n-q-j)!]}{n!}, & \text{if } 0 \le q < x \\ \frac{(n-q)!}{n!}, & \text{if } q = x. \end{cases}$$
(21)

<sup>702</sup> Then, based on (3), given the single-hop broadcast is suc-<sup>703</sup> cessful, the conditional probability that the receiver success-<sup>704</sup> fully receives the broadcast message at time slot k for both <sup>705</sup> scenarios under the QoS-based broadcast scheme,  $P_1(k)$  and <sup>706</sup>  $P_2(k)$ , can be obtained.

### 4.2.3 Single-Hop Broadcast Collision Rate for the QoS-Based Broadcast Scheme

Then, we calculate the single-hop broadcast collision rate for the QoS-based broadcast scheme. The probability that two senders have a broadcast collision is equivalent to the probability that all the common channels selected by the two senders appear in the same time slots. Therefore, using r14 (21), the probability that a single-hop broadcast fails due to r15 broadcast collisions for the QoS-based broadcast scheme is

716 
$$P_q(A, C, B) = \sum_{y=y*}^{y^{**}} \frac{\binom{Z_{AB}}{y}\binom{N_A - Z_{AB}}{n-y}\binom{Z_{ABC}}{y}}{\binom{N_A}{n}\binom{Z_{AB}}{y}^2} \frac{(n-y)!}{n!}.$$
 (22)

#### 717 4.3 Distributed Broadcast Scheme

<sup>718</sup> The third broadcast scheme considered in this paper is <sup>719</sup> called the *distributed broadcast scheme* [14],[33]. In this <sup>720</sup> scheme, all SU nodes in the network intelligently select <sup>721</sup> a subset of available channels from the original available



Fig. 10. Example of the broadcasting sequences of the distributed broadcast scheme.

channel set for broadcasting. The size of the downsized 722 available channel set is denoted as w. The value of w needs 723 to be carefully designed to ensure that at least one common 724 channel exists between the downsized available channel 725 sets of the SU sender and each of its neighboring nodes. 726 Fig. 10 gives an example of the broadcasting sequences of 727 the distributed broadcast scheme. For a SU sender, it hops 728 periodically on the w available channels for w cycles (one 729 cycle consists of  $w^2$  time slots). For each receiver, it stays 730 on one of the w available channels for w time slots. Then, 731 it repeats for every channel in the w available channels. 732

#### 4.3.1 Single-Hop Successful Broadcast Ratio for the Distributed Broadcast Scheme

Similar to the previous schemes, we first calculate the <sup>735</sup> single-hop successful broadcast ratio for the distributed <sup>736</sup> broadcast scheme. As discussed above, the size of the <sup>737</sup> downsized available channel set, w, has significant impact <sup>738</sup> on the performance of the distributed broadcast scheme. <sup>739</sup> If w is given, the single-hop successful broadcast ratio <sup>740</sup> is equivalent to the probability that the sender and the <sup>741</sup> receiver have at least one channel in common in their <sup>742</sup> downsized available channel sets. That is,  $P_{dist} = 1 -$ <sup>743</sup> Pr(Z(0, i) = 0), where Pr(Z(0, i) = 0) is the probability <sup>744</sup> that the sender and the receiver do not have any com-<sup>745</sup> mon channel in their downsized available channel sets. <sup>746</sup> The derivation process of Pr(Z(0, i) = 0) is the same as the <sup>747</sup> method proposed in [14].

#### 4.3.2 Single-Hop Average Broadcast Delay for the Distributed Broadcast Scheme

Then, we calculate the single-hop average broadcast delay 751 for the distributed broadcast scheme. For simplicity, we 752 assume that the *w* obtained by the receiver is the same as 753 the *w* of the sender. In addition, we denote the number of 754 common channels between the sender and the receiver as 755 *z*. We calculate the probability that the single-hop broadcast is successful at time slot *k* under Scenario I. Based on 757 the broadcast protocol proposed in [14], the broadcasting 758 sequence of a receiver consists of *w* sections where each 759 section includes the same channel repeated for *w* times. 760 Similar to the QoS-based broadcast scheme, the probability that for a particular *z*, the channel in  $t_{\lfloor \frac{k-1}{w} \rfloor + 1}$  is the 762 first appearing common available channel in the downsized available channel set of the sender is expressed as 764  $(w = \lfloor \frac{k-1}{w} \rfloor - 1)$ 

$$\Pr(t_{\lfloor\frac{k-1}{w}\rfloor+1}) = \frac{\binom{w-\lfloor\frac{2}{w}\rfloor-1}{z-1}}{\binom{w}{z}}.$$
765

In addition, given that the first appearing common 766 available channel is in  $(\lfloor \frac{k-1}{w} \rfloor + 1)$ -th section, the condi-767 tional probability that the broadcast is successful in time 768 slot k is  $\frac{1}{w}$ . Therefore, for Scenario I, the probability that

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 $_{769}$  the single-hop broadcast is successful at time slot k is  $_{770}$  expressed as

$$P_{I}(k) = \begin{cases} \sum_{z=1}^{w} \frac{\binom{w-\lfloor \frac{k-1}{z-1} \rfloor -1}{w\binom{w}{z}}}{w\binom{w}{z}} \Pr(z), & \text{if } k \le w(w-z) \\ \sum_{z=1}^{w} \frac{1}{w\binom{w}{z}} \Pr(z), & \text{if } w(w-z) < k \le w(w-z+1) \\ 0, & \text{if } k > w(w-z+1), \end{cases}$$

$$(23)$$

<sup>773</sup> where Pr(z) is the probability that there are *z* common chan-<sup>774</sup> nels in the downsized available channel sets between the <sup>775</sup> sender and the receiver. The derivation process of Pr(z) is <sup>776</sup> given in [14].

Then, for Scenario II, denote the numbers of common 777 778 available channels that the two senders have with the <sup>779</sup> receiver in the downsized available channel sets as  $z_1$  and  $_{780}$   $z_2$ , respectively. In addition, denote the number of channels 781 in the downsized available channel sets of the two senders 782 that are available for all three nodes as x. Since the available 783 channels are evenly distributed in the spectrum band, the 784 probability that there are x channels that are available for 785 all three nodes in their downsized available channel sets is  $_{766}$   $G(x) = {\binom{z^*}{x}} P_A^x (1 - P_A)^{z^* - x}$ , where  $P_A$  is the probability that a <sup>787</sup> channel is available for all three nodes and  $z^* = \min(z_1, z_2)$ . 788 In addition,  $P_A$  can be obtained from [14]. Therefore, simi-789 lar to the QoS-based broadcast scheme, the probability that <sup>790</sup> the single-hop broadcast is successful at time slot k under <sup>791</sup> Scenario II is expressed in (24), where U(q) is the probabil-<sup>792</sup> ity that there are q channels out of x channels appearing at <sup>793</sup> the same time slots. In addition,  $q^* = \min(x, z^* - 1)$ . Using 794 (21), U(q) can be written as

795 
$$U(q) = \begin{cases} \frac{\binom{x}{q}[(w-q)! - \sum_{j=1}^{x-q}(-1)^{(j+1)}\binom{x-q}{j}(w-q-j)!]}{w!}, & \text{if } 0 \le q < x \\ \frac{(w-q)!}{w!}, & \text{if } q = x. \end{cases}$$
(25)

<sup>796</sup> Then, based on (3), given the single-hop broadcast is suc-<sup>797</sup> cessful, the conditional probability that the receiver success-<sup>798</sup> fully receives the broadcast message at time slot *k* for both <sup>799</sup> scenarios under the distributed broadcast scheme,  $P_1(k)$  and <sup>800</sup>  $P_2(k)$ , can be obtained.

### 4.3.3 Single-Hop Broadcast Collision Rate for the Distributed Broadcast Scheme

<sup>803</sup> Finally, we calculate the single-hop broadcast collision rate <sup>804</sup> for the distributed broadcast scheme. Note that in [14], <sup>805</sup> a broadcast collision avoidance scheme is proposed. If <sup>806</sup> this scheme is used, broadcast collisions can be avoided. <sup>807</sup> However, it involves significant changes to the broadcast-<sup>808</sup> ing sequences of the senders shown in Fig. 10. To make <sup>809</sup> the analysis tractable, in this paper, we do not consider <sup>810</sup> the broadcast collision avoidance scheme. Therefore, simi-<sup>811</sup> lar to the QoS-based broadcast scheme, the probability that <sup>812</sup> a single-hop broadcast fails due to broadcast collisions for



Fig. 11. Synchronized time slots for IEEE 802.11 chipsets.

the distributed broadcast scheme is

$$P_q(A, C, B) = \sum_{z=1}^{w} \frac{(w-z)!}{w!} P_A^z \Pr(z).$$
(26) 814

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#### 5 PERFORMANCE EVALUATION

In this section, we validate our proposed unified analytical 816 model using both hardware implementation and simulation 817 in order to prove its correctness. 818

#### 5.1 Validating Analysis Using Hardware Implementation

The considered broadcast schemes have been implemented in embedded wireless radios. Each radio contains a Qualcomm Atheros IEEE 802.11 a/b/g chipset, and MADWIFI is used as the medium access control (MAC) driver. The three broadcast schemes are implemented as sub-functions of the MAC driver.

#### 5.1.1 Time Slot and Synchronization

To support synchronized transmission of broadcast messages in different time slots, we first need to implement <sup>829</sup> timing events that are synchronized among all communication nodes [34]. In order to minimize the impact by <sup>831</sup> the software in the driver, a hardware register called software beacon alert (SWBA) is utilized to generate timing <sup>833</sup> events. To support different timing events, the value in the <sup>834</sup> SWBA register must be set into the time interval between <sup>835</sup> the current timing event and the next expected timing <sup>836</sup> event. Based on this mechanism, the time-line of each communication node is split into consecutive time slots each <sup>838</sup> consisting of two portions: channel switching (CSS) and <sup>839</sup> packet transmission/reception (PTR), as shown in Fig. 11. <sup>840</sup>

To synchronize time slots among all nodes, we adopt <sup>841</sup> two mechanisms of IEEE 802.11 [35]: target beacon transmission time (TBTT) and timing synchronization function <sup>843</sup> (TSF). Within each beacon interval, the first time slot must <sup>844</sup> be aligned with TBTT, as shown in Fig. 11. Through TSF, <sup>845</sup> the time in the TSF register of different nodes is synchronized. Since TBTT is determined based on the timing value <sup>847</sup> of the TSF register, the time slots of different nodes are <sup>848</sup> synchronized accordingly. <sup>849</sup>

$$P_{II}(k) = \begin{cases} \sum_{z_1=1}^{w} \sum_{z_2=1}^{w} \sum_{x=0}^{z^*} \sum_{q=0}^{q^*} \frac{\binom{w-\lfloor \frac{k-w}{w} \rfloor -1}{z_1+z_2-2q-1}}{w(z_1+z_2-2q)} \Pr(z_1)\Pr(z_2)G(x)U(q), & \text{if } k \le w(w-z_1+z_2+2q) \\ \sum_{z_1=1}^{w} \sum_{z_2=1}^{w} \sum_{x=0}^{z^*} \sum_{q=0}^{q^*} \frac{1}{w(z_1+z_2-2q)} \Pr(z_1)\Pr(z_2)G(x)U(q), & \text{if } w(w-z_1+z_2+2q) < k \le w(w-z_1+z_2+2q+1) \\ 0, & \text{if } k > w(w-z_1+z_2+2q+1). \end{cases}$$
(24)



### 5.1.2 Packet Transmission/Reception and Channel Selection

852 In a source node, a broadcast message is generated in 853 the PTR portion of a time slot and is then sent in a 854 selected channel. This process repeats for S time slots. Other 855 nodes in the network attempt to receive the broadcast mes-856 sage from its neighboring nodes and then rebroadcast it. 857 Due to slot-by-slot operation, when a broadcast message 858 is received, it is rebroadcast in the next time slot in the 859 selected channel. This process is also repeated for S time 860 slots. Since the same message may be received for multi-861 ple times, a sequence number is added into each broadcast <sup>862</sup> message to avoid redundant broadcast messages. It should 863 be noted that the channel selection for packet transmission 864 and reception follows the rules set by the specific broad-865 cast schemes developed in this paper. The channel set in 866 each node reflects the activities of primary nodes and is 867 determined according to off-line simulations.

#### 868 5.1.3 Performance Measurement

<sup>869</sup> Two performance metrics are used in our implementation: <sup>870</sup> the successful broadcast ratio and the average broadcast <sup>871</sup> delay. The former metric measures the probability that a <sup>872</sup> broadcast message can be successfully received by all nodes <sup>873</sup> in a network, and the latter one records the average delivery <sup>874</sup> time from the source node to the last node. In order to get <sup>875</sup> stable performance results, we repeat the experiments for <sup>876</sup> *N* measurements as shown in Fig. 12. Within  $t_e$  seconds, <sup>877</sup> one round of experiment is conducted.  $t_e$  is selected large <sup>878</sup> enough so that all non-source nodes finish the process of <sup>879</sup> receiving/rebroadcasting messages within the same period. <sup>880</sup> In our experiments, we set  $t_e$  to be 3 seconds for a multi-<sup>881</sup> hop CR ad hoc network under Topology 1 as shown in <sup>882</sup> Fig. 13(a).

Fig. 14 shows comparisons between analytical results and experimental measurements for the random and QoSbased broadcast schemes. The comparisons for the distributed broadcast scheme are depicted in Fig. 15, where two cases are considered: 1) Case 1: all nodes have the same w (i.e., w(A) = w(B) = w(C) = w(D) = 5) and 2) Case 2: some nodes have different w (i.e., w(A) = w(B) =



Fig. 13. Topology 1 and 2 considered in the performance evaluation. (a) Topology 1. (b) Topology 2.



Fig. 14. Analytical and implementation results using the random and QoS-based broadcast schemes under Topology 1. (a) Successful broadcast ratio. (b) Average broadcast delay.

w(D) = 5 and w(C) = 4). As we can see from Figs. 14 890 and 15, the implementation results fit the analytical results 891 fairly well. 892

#### 5.2 Validating Analysis Using Simulation

Due to the constraint on the total number of channels for 894 hardware testing, we also use simulations to validate our 895 proposed analytical model when the number of channels 896 varies from 10 to 40. The side length of the simulation area 897  $L_s=10$  (unit length). PUs are evenly distributed within this 898 area. The total number of PUs is denoted as K = 40. The 899 total number of channels is denoted as M. Furthermore, 900 each SU has a circular transmission range with a radius 901 of  $r_c$ . The SUs within the transmission range are consid-  $_{902}$ ered as the neighboring nodes of the corresponding SU. In 903 addition, each SU also has a circular sensing range with 904 a radius of  $r_s$ . That is, if a PU is currently active within 905 the sensing range of a SU, the corresponding SU is able to 906 detect its appearance. Moreover, we consider the PU traf- 907 fic model used in [36], where the PU packet inter-arrival 908 time follows the biased-geometric distribution [37],[38]. In 909 fact, our proposed algorithms do not rely on specific PU 910 traffic models. We assume that the probability that a PU 911 is active is fixed (i.e.,  $\rho = 0.9$ ). Each PU randomly selects 912 a channel from the spectrum band to transmit one packet. 913 Since the available channels for each SU depends on the 914 sensing outcome in its sensing range, we use the values 915 from the simulation as the input for the proposed analyti- 916 cal model (e.g., the number of common available channels 917 between nodes A and B,  $Z_{AB}$ ). In addition, we assume that 918 the SU channel availability is stable during a broadcast 919 duration. 920



Fig. 15. Analytical and implementation results using the distributed broadcast scheme under Topology 1. (a) Successful broadcast ratio. (b) Average broadcast delay.



Fig. 16. Analytical and simulation results of the single-hop successful broadcast ratio using the three broadcast schemes under Scenario I and II. (a) Random broadcast scheme. (b) QoS-based broadcast scheme. (c) Distributed broadcast scheme.



Fig. 17. Analytical and simulation results of the single-hop average broadcast delay using the three broadcast schemes under Scenario I and II. (a) Random broadcast scheme. (b) QoS-based broadcast scheme. (c) Distributed broadcast scheme.

#### 921 5.2.1 Single-Hop Performance

922 We first investigate the single-hop performance of each 923 broadcast protocol considered in this paper, because this 924 performance is the foundation of the multi-hop perfor-925 mance evaluation. We study the two single-hop broadcast scenarios shown in Fig. 7. In our study, the nodes are at 927 the border of each other's sensing range. Fig. 16(a) to (c) 928 show the analytical and simulation results of the single-929 hop successful broadcast ratio using the three considered 930 broadcast schemes under Scenario I and II. For the random  $_{931}$  broadcast scheme,  $S_r$  is set to be the same as the num-932 ber of channels, M. For the QoS-based broadcast scheme,  $_{933}$  n = 2 and S = 2M. In addition, for the distributed scheme,  $_{934} w = 5$ . It is shown that the simulation and analytical 935 results match very well with the maximum difference of 936 0.4%, 0.5%, and 0.7% for the three schemes, respectively. <sup>937</sup> The figure indicates that the distributed broadcast scheme 938 can achieve the highest single-hop successful broadcast 939 ratio.

In addition, Fig. 17(a) to (c) illustrate the analytical and simulation results of the single-hop average broadcast delay using the three considered broadcast schemes under Scenario I and II. It is also shown that the simulation and analytical results match very well with the maximum difference of 1.4%, 3.7%, and 5.5% for the three schemes, even the distributed broadcast scheme results in the at lowest single-hop average broadcast delay among the three set schemes.

#### 5.2.2 Successful Broadcast Ratio of Multi-hop CR Ad Hoc Networks

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Next, we investigate the multi-hop performance. For 951 the successful broadcast ratio, we study the two 952 topologies shown in Fig. 13(a) and (b). The coordi- 953 nates of nodes in Topology 1 are A(4, 4), B(6, 4), C(5, 2.28), 954 and D(7, 2.28). On the other hand, note that Topology 955 2 is a 6-node network under arbitrary topology. 956 Moreover, the coordinates of nodes in Topology 2 are 957 A(4, 4), B(5.8, 4.8), C(5, 3), D(6.6, 3), E(7, 4.5), and F(3, 5). 958 The parameters of each broadcast scheme are set to be the 959 same as in the single-hop performance evaluation. In all 960 topologies considered in the performance evaluation, node 961 A is the source node. Fig. 18(a) to (c) show the analytical  $_{962}$ and simulation results of the broadcast ratio using the 963 three considered broadcast schemes under Topology 1 and 964 2. It is shown that the simulation results fit the analytical 965 results well with the maximum difference of 2.1%, 4.6%, 966 and 0.4% for the three schemes, respectively. The dis-967 tributed broadcast scheme still has the best performance 968 of successful broadcast ratio among the three schemes. 969

#### 5.2.3 Average Broadcast Delay of Multi-hop CR Ad Hoc Networks

For the average broadcast delay, we investigate two grid  $_{972}$  topology networks: 1) a 3 × 3 grid network (denoted as  $_{973}$  Topology 3); and 2) a 4 × 4 grid network (denoted as  $_{974}$ 



Fig. 18. Analytical and simulation results of the successful broadcast ratio using the three broadcast schemes under Topology 1 and 2. (a) Random broadcast scheme. (b) QoS-based broadcast scheme. (c) Distributed broadcast scheme.



Fig. 19. Analytical and simulation results of the average broadcast delay using the three broadcast schemes under Topology 3 and 4. (a) Random broadcast scheme. (b) QoS-based broadcast scheme. (c) Distributed broadcast scheme.

<sup>975</sup> Topology 4). Fig. 19(a) to (c) depict the analytical and simu-<sup>976</sup> lation results of the average broadcast delay using the three <sup>977</sup> considered broadcast schemes under Topology 3 and 4. It <sup>978</sup> is shown that the simulation and analytical results coincide <sup>979</sup> with each other well with the maximum difference of 4.9%, <sup>980</sup> 9.4%, and 6.5% for the three schemes, respectively. Again, <sup>981</sup> the distributed broadcast scheme has a much lower average <sup>982</sup> broadcast delay, as compared to the other two schemes.

### 5.3 System Parameter Design Using the Proposed Analytical Model

985 As explained in Section 1, the system parameters of the 986 proposed broadcast protocols in [11]-[14] are not designed 987 to achieve the optimal performance due to the lack of 988 analytical analysis. In this paper, we investigate the sys-989 tem parameter design of the random broadcast scheme <sup>990</sup> using the proposed analytical model. In the random broad-<sup>991</sup> cast scheme, the length of time slots that the sender uses  $_{992}$  for broadcasting,  $S_r$ , is crucial to the performance of the 993 broadcasting. Note that there exists a trade-off when deter-<sup>994</sup> mining  $S_r$ . If  $S_r$  is large, the successful broadcast ratio is 995 high. However, the average broadcast delay is also long. 996 On the other hand, if  $S_r$  is small, the average broadcast 997 delay is short. However, the successful broadcast ratio is <sup>998</sup> low. Hence, to design an optimal  $S_r$  is essential to the 999 performance of the random broadcast scheme. We use an 1000 example to illustrate the process of the system parameter 1001 design. Consider a CR ad hoc network under Topology 1 1002 shown in Fig. 13(a). We assume that the single-hop success-1003 ful broadcast ratio over each link is the same, which can be

obtained from (13) (denoted as *p*). Thus, using the proposed 1004 algorithm for calculating the successful broadcast ratio, the 1005 successful broadcast ratio for the random broadcast scheme 1006 under Topology 1 is 1007

$$P_{succ} = p[1 - (1 - p)^{2} - P_{q}]^{2} + p^{3}\{1 - [1 - (1 - p)^{2} - P_{q}]\} + (1 - p)p^{2}[1 - (1 - p)^{2} - P_{q}] + (1 - p)^{2}p^{3},$$
(27) 1008

where  $P_q$  is given in (16). It is known that  $P_{succ}$  is a function 1009 of  $S_r$ .

On the other hand, we calculate the average broadcast 1011 delay under Topology 1, where node *A* is the source node. 1012 Since there are two levels in the network, we need to obtain 1013 the average broadcast delay of each level. Thus, using the 1014 proposed algorithm for calculating the average broadcast 1015 delay, we have 1016

$$\Gamma = \sum_{d=1}^{S_r} dP_1(d) + \sum_{d=1}^{S_r} dP_2(d), \qquad (28) \text{ 1017}$$

where  $P_1(d)$  and  $P_2(d)$  can be obtained from Section 4.1.2 <sup>1018</sup> and (3). Note that  $\Gamma$  is also a function of  $S_r$ . Define the objec- <sup>1019</sup> tive function of a broadcast protocol,  $\Theta$ , as the rate between <sup>1020</sup> the successful broadcast ratio and the average broadcast <sup>1021</sup> delay. Therefore, we have  $\Theta = \frac{P_{succ}}{\Gamma}$ . Thus, the optimization <sup>1022</sup> problem of the protocol design becomes finding the opti- <sup>1023</sup> mal  $S_r$  that maximizes the objective function,  $\Theta$ . Then, using <sup>1024</sup> certain numerical method, the optimal  $S_r$  can be obtained. <sup>1025</sup> Fig. 20 shows the numerical results of the objective funct- <sup>1026</sup> tion under various  $S_r$ . It is shown that a proper  $S_r$  exists <sup>1027</sup>



Fig. 20. Numerical results of the objective function under various  $S_r$ .

<sup>1028</sup> to achieve the optimal performance of a broadcast proto-<sup>1029</sup> col. For instance, when M = 10, the optimal  $S_r$  is 11. The <sup>1030</sup> corresponding successful broadcast ratio is 81.25% and the <sup>1031</sup> average broadcast delay is 8.85 time slots.

#### 1032 6 CONCLUSION

1033 In this paper, the performance analysis of broadcast pro-1034 tocols for multi-hop CR ad hoc networks is studied. Due 1035 to the non-uniform channel availability in CR networks, 1036 several significant differences and unique challenges are 1037 introduced when analyzing the performance of broadcast 1038 protocols in CR ad hoc networks. A novel unified analytical 1039 model is proposed to address these challenges and ana-1040 lyze the broadcast protocols in CR ad hoc networks with 1041 any topology. Specifically, two algorithms are proposed to 1042 calculate the successful broadcast ratio and the average 1043 broadcast delay of a broadcast protocol. In addition, the 1044 derivation methods of the single-hop performance metrics 1045 for three different broadcast protocols in CR ad hoc net-1046 works under practical scenarios are proposed. Results from 1047 both the hardware implementation and software simulation <sup>1048</sup> validate the analysis well. To the best of our knowledge, this 1049 is the first analytical work on the performance analysis of 1050 broadcast protocols for multi-hop CR ad hoc networks.

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